647510 **BSCmaMD201x-1C** **Seat No :________**

B.Sc. Semester – 1 (NEP-2020) Examination
January-2024
Matrices and Coordinate Geometry (MDC)

Time: 2:00 Hours **Marks: 50**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one.

(1) Define: Trace of a matrix, identity matrix, triangular matrix, transpose of a matrix, Nilpotent matrix. (05)

(2) in usual notation Prove that $(A+B)^T=A^T+B^T$ (05)

Que.1(B) Answer any one.

(1) Prove that every square matrix can be expressed uniquely as the sum of a symmetric matrix and skew Symmetric matrix. (05)

(2) If A is n-square matrix then prove that $A\cdot(\text{adj } A)= |A|\cdot I$ (05)

Que.2(A) Answer any one.

(1) Define: Rank of matrix and using echelon matrix. Find rank of matrix (05)

$$\begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 8 & 8 \end{bmatrix}$$

(2) Show that the row rank of a matrix is the same as its rank. (05)

Que.2(B) Answer any one.

(1) Find rank of matrix $\begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 8 & 8 \end{bmatrix}$ by represent it to normal form. (05)

(2) Prove that the rank of the product of two matrices cannot exceed the rank of either matrix. (05)

Que.3(A) Answer any one.

(1) Prove that every square matrix satisfies its characteristic equation. (05)

(2) If following equations have non zero solution then Find value of k. (05)

$x+2y+3z=kx$
$3x+y+2z=ky$
$2x+3y+z=kz$

Que.3(B) Answer any one.

(1) Find eigen values and eigen vectors of matrix $\begin{bmatrix} -3 & -6 & 7 \\ 1 & 1 & -1 \\ -4 & -6 & 8 \end{bmatrix}$ (05)

(2) Find inverse of matrix $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$ using Cayley-Hamilton theorem. (05)

Que.4(A) Answer any one.

(1) if $P^{'}P$ and $Q^{'}Q$ are the perpendicular focus chord of a conic then prove that (05)

$\frac{1}{SP.SP^{'}} + \frac{1}{SQ.SQ^{'}} =$ constant.

(2) Derive relation between Cartesian coordinates and polar coordinates. (05)

Que.4(B) Answer any one.

(1) Obtain the equation of the circle whose center is (P, α) and radius a. (05)

(2) Find the polar coordinates for the Cartesian coordinates (1,1) and the Cartesian coordinates for the polar coordinates $(1, \frac{\pi}{2})$. (05)

Que.5(A) Answer any one.

(1) If AB=BA and $S^2=B$ then show that $(A^{-1}SA)^2=B$, where A and B be two conformable matrices for multiplication. (05)

(2) If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$. Where $n \in N$. (05)

Que.5(B) Answer any one.

(1) If A= $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then Express $2A^5-3A^4+A^2-4I$ as a linear polynomial. (05)

(2) Obtain the equation of circle whose end points of diameter are (2,0) and $(3, \frac{\pi}{2})$. (05)

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